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Cascade Formation Control for Multiple Nonholonomic Mobile Robots

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Abstract: For the problem of multiple mobile robots formation control, a control strategy which consists of three methods, namely Virtual Robot tracking, $l-\varphi$ and $l-l$ control is proposed. By this method, collision avoidance and formation are achieved. However, depending on the initial distribution of robots, multiple collision detections are occurred and it turns out non-smooth convergence. In this paper, we propose a new control design which organizes some platoons of robots and leads platoons to formation goal without any collision. Then, formation with smooth motion can be established. Our technique is demonstrated in Numerical simulation.

Keywords: nonholonomic constraint, feedback, collision, virtual robot tracking control

I. INTRODUCTION

Recently, the motion planning for multiple mobile robots has been subject of a considerable research effort. Especially, the formation control for multiple mobile robots is one of the most important issues because if the robots be able to form the arbitrary formation, they are capable of performing many applications that single robots cannot.

In this paper, we focus on the formation control for multiple mobile robots with two wheels driven independently. This type robot is an underactuated mechanical system and has a non-holonomic property, and thus control problems for this system become more complex than the case with the omnidirectional mobile robot. However, it is very practical to treat nonholonomic mobile robots since many mobile robots in the real world have two, three or four wheels and have same non-holonomic constraint.

For the formation control problem of multiple 2-Wheels mobile robots, a tracking control method using the deviation model has been proposed [1]. In addition, it is necessary to consider the collision avoidance problem among the robots. Jongusuk and Mita [2] have introduced hybrid control strategy for tracking control of multiple mobile robots using virtual robots (VR) tracking control combined with *l-l* control, by Densai et al [3], in an obstacle-free environment. VR tracking control establishes the formation when no collision is detected; where VR is an ideal robot fixed with each follower and the formation is established by conforming VR to the reader robot. During transient state of this control, if a possibility of collision is detected, the control method switches to *l-l* control which is the collision avoidance technique. For more than three mobile robots, a control strategy which is expansion of previous method; namely added $l-\varphi$ control for collision avoidance, is proposed [4]. By this strategy, formation control with collision avoidance can be achieved for more than three mobile robots. However, depending on the initial distribution and attitude of each robot, there are possibilities of multiple collisions detection near the formation goal and the controller must be switched accordingly. As the result, it turns out non-smooth convergence.

In this paper, we propose a new control strategy which is named "Cascade Formation control". Our control concept is to reduce the collision avoidance behaviors (namely, $l-\varphi$, or *l-l* control) as far as possible. First, all follower robots organize some platoons at arbitrary place where the platoons don't collide with each others. And then, platoons are lead to formation goal without any collisions. By using our strategy, it is expect to establish the formation with more smooth and more safety.

II. PROBLEM FORMULATION

1. Two Wheels Mobile Robot

A 2-Wheel mobile robot, which kinematic model is defined by

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \qquad (1)$$

where, (x_i, y_i) denotes the control point of in global frame and θ_i is the orientation; and $[v_i, \omega_i]^T$ is the control input, with v_i is the translational velocity and ω_i is the angular velocity of robot i. In addition, the robot satisfy non-holonomic velocity constraints, which encompass pure-rolling and non-slipping conditions,

non-slipping : $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$ (2)

pure rolling : $\dot{x}\cos\theta + \dot{y}\sin\theta = v$ (3)

2. Assumptions

I. Robots are of the same model and satisfy non-slipping and pure rolling constraints.

II. The workspace is flat and contains no obstacle.

III. The reference robot follows a smooth trajectory and maintains positive velocity.

IV. Each follower robot is indexed by a distinctive priority number and aware of other's indexes.

V. Each robot can extract necessary information vi a its communication equipment.

3. Problem statement

Giving initial positions and orientations of the follower robots and the motion of the reference robot, the objective is to design for follower i-th robots such that as $t \to \infty$,

1. Formation is established.
2. No collision among robot i and robot j.

III. Cascade Formation Control

Under the assumption in the last section, Jongusuk and Mita [2] and Ikeda et al [4] have achieve the formation control with collision avoidance for three or more mobile robots by using virtual robots (VR) tracking control combined with *l-l* control and $l-\varphi$ control. However, depending on the initial distribution and attitude of each robot, there is a possibility of multiple collisions detection near the reference formation as Fig.1.

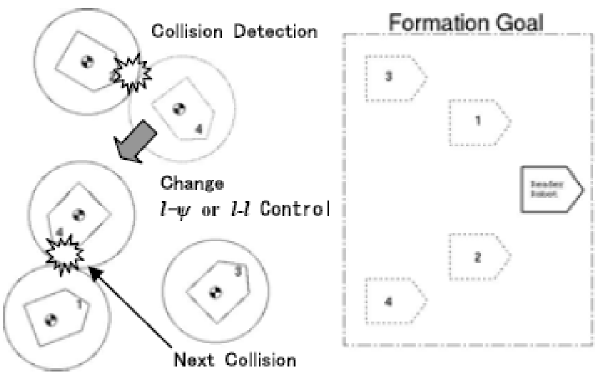

Fig.1. Multiple Collisions Detection

Where, the circles centering at the control points of robots are the required clearances between robots. Each robot detects the collision by checking its circle touch the other circles, and changes the control to $l-\varphi$ or $l-l$ control if its priority number is lower than other's. By these controls, lower priority robots take avoidance behaviors. However, in the case that the positions of robots are closely, the collision avoidance behaviors cause other new collision possibilities. Therefore, it turns out non-smooth convergence.

For this problem, we propose a control strategy which reduces the collision avoidance behaviors as far as possible. This strategy divides all follower robots into some groups and robots of each group organize a platoon at arbitrary place. Where, platoons don't collide with other platoons. And then, platoons are leaded to formation goal without any collisions. We call this control strategy, "Cascade Formation Control". Fig.2 is the schematic diagram of our idea.

Remark: In this paper, we teat the formation control problem for 1 reader robot and 4 follower robots like Fig.2 in order to simplify the problem and description.

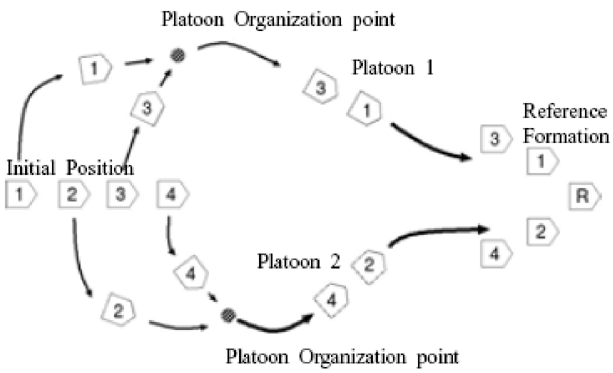

Fig.2. Cascade Formation Control

For the platoon organization, it has to be decided the number of platoons, member robots and location. In the case of our supposition, due to the number of all follower robots is four and composition of the formation goal (see Fig.2), we decide to organize two platoons. The members of "Platoon 1" are follower robot 1 and robot 2, and the members of "Platoon 2" are robot 2 and robot 4, respectively.

The location where robots organize the platoon should be decided to be that the follower robots do not detect the collision as far as possible during organizing platoons and platoons don't collide with others until the formation is established. Although it may be that there are many procedures to decide it and it become a new problem of how to decide the location, we simply decide it from the initial positions of follower robots and the formation goal.

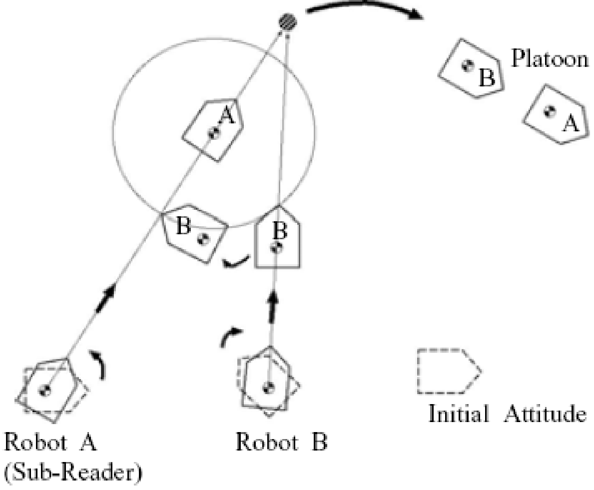

Fig.3 Platoon Organization

Fig.3 shows one platoon organization for two robots. The robot which has higher priority number is selected as a reader of the platoon. We call it "Sub-Reader". In Fig3, Robot A is Sub Reader. The sequence of platoon organization is described as follows.

1. Each robot turns to the platoon organization point (P.O.P) at the initial position.
2. Each robot goes to the P.O.P.
3. If robot B touches the circle of robot A, the control of robot B switches to the VR control to follow the robot A.

In item 2, each robot goes to the P.O.P by using the tracking control method of [1]. In this method, a reference trajectory is given as the wheel ruts, namely it has the same kinematics model of equation (1). For the reference velocities $[\ v_r, \omega_r\ ]^T$, reference posture $[\ x_r, y_r, \theta_r\ ]^T$ is decided and the tracking control is achieved by stabilizing the error system

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega_r y_e \\ v_r \sin\theta_e - \omega_r x_e \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -y_e \\ 0 & x_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \qquad (4)$$

where, x_e, y_e, θ_e are derived as

$$\begin{aligned} x_e &= (x_r - x)\cos\theta + (y_r - y)\sin\theta \\ y_e &= -(x_r - x)\sin\theta + (y_r - y)\cos\theta \\ \theta_e &= \theta_r - \theta \end{aligned} \qquad (5)$$

and μ_1, μ_2 are new inputs for the error system (4) which are designed based on the Liapunov stabilization theory.

Thus we have to decide the reference velocities for each robot to reach the P.O.P. By item 1, it becomes easy to decide the reference velocities since it is not necessary to decide the angular velocity ω_r. As the result, trajectories become to straight lines. Turn motion in item 1 is achieved by the feed forward control.

In item 3, the platoon is established by VR control. The concept of VR is used to avoid collisions between the follower robots and the reference robot. VR is a hypothetical robot whose orientation is identical to that of its corresponding follower robot, but position is placed apart from by the predefined *r*-*l* clearances.

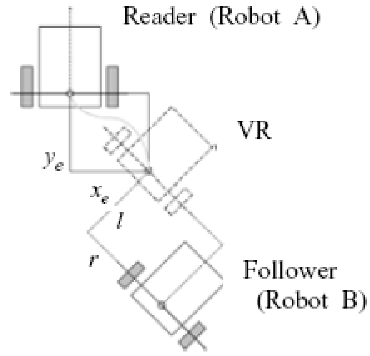

Fig.4 VR tracking model

The relation between VR and the follower robot B is written as follows.

$$\begin{bmatrix} x_{vB} \\ y_{vB} \\ z_{vB} \end{bmatrix} = \begin{bmatrix} x_B - r\sin\theta_B + l\cos\theta_B \\ y_B + r\cos\theta_B + l\sin\theta_B \\ \theta_B \end{bmatrix} \qquad (6)$$

Where, $[x_B, y_B, \theta_B\ ]^T$ refers to the follower robot B and $[x_{vB}, y_{vB}, \theta_{vB}\ ]^T$ is for VR of follower robot B. The kinematic model of VR is then.

$$\begin{bmatrix} \dot{x}_{vB} \\ \dot{y}_{vB} \\ \dot{z}_{vB} \end{bmatrix} = \begin{bmatrix} \cos\theta_B & -r\cos\theta_B - l\sin\theta_B \\ \sin\theta_B & -r\sin\theta_B + l\cos\theta_B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_B \\ \omega_B \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{vB} \\ 0 \quad 1 \end{bmatrix} \mathbf{u}_B \quad (7)$$

The idea of VR tracing is to use VR to track the reference robot, then the follower will approach the desired position in the platoon as its VR approaches the reference robot. In Fig.4, it is shown that the VR approaches the reference robot in a internal shape x_e and y_e. Details about the controller can be found in [2].

By using VR control, if the radius of circle of A (in Fig.3) is greater than the clearance between robot A and robot B ($\sqrt{l^2+r^2}$), there is no collision during transient state of platoon organization.

Finally, when the sub-reader of the Platoon arrives in P.O.P, sub-reader switches control to the VR control for Reader Robot and the formation will be established.

IV. NUMERICAL SIMULATION

Fig.5 shows the simulation results for our techniques. In these figures, F1~F5 denotes the follower robots and the priority number. The reference formation is same as Fig.1 or Fig.2. In second figure, robots turn to the P.O.P to organize two platoons, where, F1&F3, and F2&F4 are members of each platoon, respectively. F1 and F2 play sub-reader and in third figure, F3 and F4 switch the control to the VR control to follow each sub-reader. When sub-readers arrive in the P.O.P, they switch the control to the VR control to follow the reader robot as fourth figure. Finally, the formation is established in bottom figure.

As a result, the formation control can be established without any collision by using our strategy.

V. CONCLUSION

In this paper, we have presented a new control strategy for controlling multiple mobile robots (1-reader and 4-followers case). The effectiveness of our strategy is demonstrated in Numerical simulation.

REFERENCES

[1] Y. Kanayama, Y. Kimura, F. Miyazaki et al (1990), Tracking Control Method for an Autonomous Mobile Robot, IEEE Int. Conf. on Robotics & Automation, pp.384-389
[2] J. Jongusuk and T. Mita (2001), Tracking Control of Multiple Mobile Robots: A Case Study of Inter-Robot Collision-Free Problem, Proceedings of 2001 IEEE Intl. Conf on Robotics & Automation, pp.2885-2890
[3] J. P. Densai, J. Ostrowski, V. Kurma (1998), Controlling Formations of Multiple Mobile Robots, IEEE Proceeding of Intl. Conf. Robotics and Automations, pp.2864-2869
[4] T Ilkeda, J. Jongusuk, T Ikeda, et al (2004), Formation Control of Multiple Nonholonomic Mobile Robots (in Japanese), IEEEJ Trans. IA, Vol.124, No8, pp.814-819

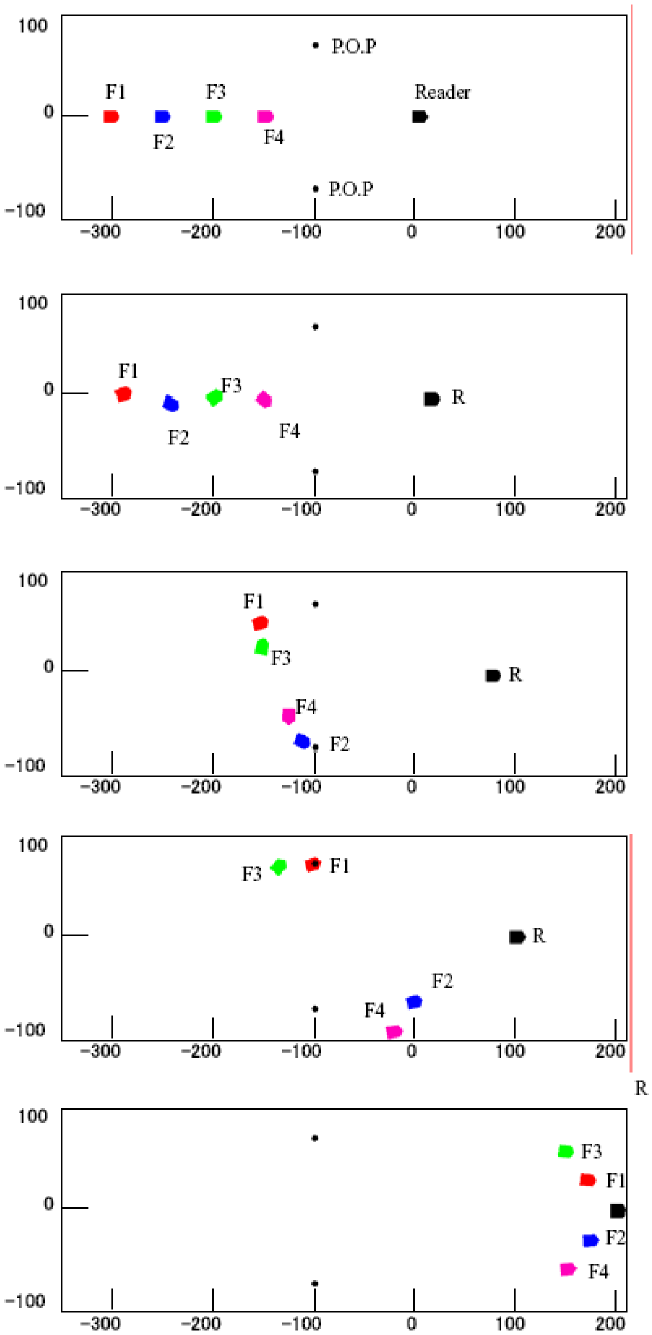

Fig.5 Simulation Result